GAMMA-RAY BURST BEAMING: A UNIVERSAL CONFIGURATION WITH A STANDARD ENERGY RESERVOIR?

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ABSTRACT

We consider a gamma-ray burst (GRB) model based on an anisotropic fireball with an axisymmetric energy distribution of the form $\epsilon(\theta) \propto \theta^{-k}$, and allow for the observer's viewing direction being at an arbitrary angle θ_v with respect to the jet axis. This model can reproduce the key features expected from the conventional on-axis uniform jet models, with the novelty that the achromatic break time in the broadband afterglow lightcurves corresponds to the epoch when the relativistic beaming angle is equal to the viewing angle, θ_v, rather than to the jet half opening angle, θ_j. If all the GRB fireballs have such a similar energy distribution form with $1.5 < k \lesssim 2$, GRBs may be modeled by a quasi-universal beaming configuration, and an approximately standard energy reservoir. The conclusion also holds for some other forms of angular energy distributions, such as the Gaussian function.

Subject headings: gamma rays: bursts - shock waves - ISM: jets and outflows

1. INTRODUCTION

Recently, several independent approaches have led to the conclusion that long gamma-ray bursts (GRBs) have a standard energy reservoir of several 10^{50} ergs (Frail et al. 2001, hereafter F01; Panaitescu & Kumar 2001, hereafter PK01; Piran et al. 2001, hereafter P01). An important ingredient of this argument is that the putative jet opening angles, θ_j, as inferred from the afterglow lightcurve breaking times, t_b, have a broad distribution, but just of the right form to compensate for the wide dispersion of the "isotropic" energy emitted in γ-rays, $E_{\gamma,\mathrm{iso}}$, so that $E_\gamma = (E_{\gamma,\mathrm{iso}}/4\pi)(\theta_j^2/2)$ is essentially invariant (F01). The total energy of the fireball should be $E_{\mathrm{tot}} \geq E_\gamma + E_0$, where E_0 is the initial kinetic energy of the fireball in the afterglow phase assuming an adiabatic evolution, and the inequality takes into account the possible energy loss during the radiative regime in the early afterglow phase that has evaded the present observations, as well as energy losses outside the γ-ray band (e.g. the BATSE window) or in non-electromagnetic forms (e.g. neutrinos and gravitational waves) during the prompt phase. Writing $E_\gamma = \eta E_{\mathrm{tot}}$ where η is the gamma-ray emission efficiency, E_{tot} could be mainly contributed by E_0 if η is small (e.g. < 0.1). PK01 and P01 found that E_0 is also distributed in a narrow range. For a uniform jet, this leads to the inference that $E_0 = (dE/d\Omega)(\theta_j^2/2)$ is also essentially invariant. However, in the above analysis, and in the current afterglow jet models which are used to determine θ_j, it is generally assumed that the jets are uniform, with sharp cut-offs at the edges, and that the line-of-sight cuts right across the jet axis. None of these assumptions are necessarily true in general (Mészáros, Rees & Wijers 1998; MacFadyen & Woosley 1999; Woods & Loeb 1999; Nakamura 1999; Paczyński 2001; Salmonson 2001; Dai & Gou 2001). On the other hand, although it is not difficult to construct a central engine model which makes GRBs with a standard energy reservoir but with quite different beaming angles, it would be more elegant to have a model that all the GRB beams share a standard energy reservoir as well as *a quasi-*

universal beaming configuration (M. J. Rees, 2001, private communication). Here we show that such a model can be constructed by taking account of the off-axis anisotropic jet effects, without violating the present observational constraints.

2. THE MODEL

Our assumption is that all the long GRBs have a quasi-universal beam configuration, with a strong anisotropy of the angular distribution of the fireball energy around an axial symmetry. The jet axis is physically related to the rotational axis of the central engine, so it is reasonable to assume that initially the closer to the jet axis, the higher the energy concentration. The actual angular distribution of the fireball energy is unknown, and we model it as (e.g. Mészáros et al. 1998)

$$dE/d\Omega = \epsilon(\theta, \phi) = \epsilon(\theta) = \epsilon_0 \theta^{-k}, \qquad (1)$$

within the range $\theta_m \leq \theta \leq \Theta$, where θ_m is a very small angle within which some deviation from (1) is necessary to avoid the divergence at $\theta = 0$, and Θ is some large angle which exceeds the presently measured θ_j by at least a factor of two (for the simplification of the discussions below). The real angular energy distribution may differ from the power law (1), but most of our discussions below can be generalized to other forms of distribution functions (e.g. see [6] and relevant discussions below). The adoption of (1) is for the simplicity of the discussions. The angular-dependence of the baryon loading rate is uncertain, and we assume that it is weak so that the Lorentz factor angular distribution follows a similar law, i.e., $\Gamma(\theta) \propto \theta^{-k}$ (of course, the law should be modified when $\Gamma(\theta)$ approaches unity). We make furthermore the assumption that

$$(1.5) < k \lesssim 2 \qquad (2)$$

in (1). The reason for this requirement will become evident later. The main conjecture of the model is that *the dispersion in the afterglow data of the breaking time, t_b, is a manifestation of the diversity of viewing angles of the*

observers, rather than to the diversity of intrinsic opening angles of the jets themselves. In other words, that what were inferred by Frail et al. (2001) as θ_j are essentially θ_v in our model, where θ_v is the observer's viewing angle with respect to the jet axis. We will test whether the above hypothesis is able to pass the following three criteria: (i) When $\Gamma(\theta_v) \gg 1/\theta_v$, the jet dynamics along the line-of-sight satisfies the isotropic law $\bar{\Gamma}(\theta_v, t) \propto t^{-3/8}$ (for simplicity, we only discuss an adiabatic fireball running into an interstellar medium with a constant density), where t is the observer time, and $\bar{\Gamma}(\theta_v, t)$ is an effective Lorentz factor assuming an isotropic fireball which could mimic the emission in the direction θ_v at the time t; (ii) When $\Gamma(\theta_v) \lesssim 1/\theta_v$, the dynamics changes so that the lightcurves steepen; (iii) The total jet energy E_{tot} is essentially a universal value.

For an isotropic adiabatic fireball running into a uniform medium, $\Gamma(t) \propto \epsilon^{1/8} n^{-1/8} (at)^{-3/8}$, where $\epsilon = dE/d\Omega$ is the energy per solid angle, n is the ambient medium number density, and the blastwave radius is written in a general form as $R = a\Gamma^2 ct$, where the factor a effectively takes into account the surface of equal-arrival-time as well as the thickness of the emitting region. To test the criteria (i), the key is to estimate the effective energy per solid angle, $\bar{\epsilon}(\theta_v, \phi_v, t) = \bar{\epsilon}(\theta_v, t)$, in the direction (θ_v, ϕ_v), and to evaluate the possible time-dependence of this value. When $\Gamma(\theta_v, t) = \Gamma \gg 1/\theta_v$, the observer can only observe a solid angle around (θ_v, ϕ_v) with a half opening angle of order $1/\Gamma$ due to the relativistic beaming effect[1]. By definition, the effective energy per solid angle in the direction (θ_v, ϕ_v) is

$$\bar{\epsilon}(\theta_v, \phi_v, t) = \bar{\epsilon}(\theta_v, t) = \frac{\int_{\theta_v - 1/\Gamma}^{\theta_v + 1/\Gamma} \epsilon(\theta, t) \sin\theta d\theta}{\int_{\theta_v - 1/\Gamma}^{\theta_v + 1/\Gamma} \sin\theta d\theta} \quad (3)$$

due to the axial symmetry. In the small angle approximation, which is relevant to the present discussions[2], one has $\sin\theta \sim \theta$. When $\Gamma \gg 1/\theta_v$ and noticing (1), this gives

$$\bar{\epsilon}(\theta_v, t) \simeq \frac{\int_{\theta_v - 1/\Gamma}^{\theta_v + 1/\Gamma} \epsilon_0 \theta^{1-k} d\theta}{\int_{\theta_v - 1/\Gamma}^{\theta_v + 1/\Gamma} \theta d\theta} \simeq \epsilon_0 \theta_v^{-k} = \epsilon(\theta_v). \quad (4)$$

This is a time-independent quantity, since the sideways expansion effect is not important at the same stage (see discussions below). We then have

$$\bar{\Gamma}(\theta_v, t) \propto [\bar{\epsilon}(\theta_v, t)]^{1/8} n^{-1/8} (at)^{-3/8} \propto t^{-3/8}, \quad \Gamma \gg 1/\theta_v. \quad (5)$$

This indicates that the observer does not feel the anisotropy of the fireball when the relativistic beaming angle $1/\Gamma$ is much smaller than the viewing angle θ_v, but observes the fireball as if it were isotropic. This is the same conclusion as drawn in the on-axis uniform jet model. The conclusion (4) does not require (2) and holds for any k value. In fact, it even holds for some other forms of angular energy distributions, e.g. the Gaussian distribution,

$$\epsilon(\theta) = \epsilon_0 \exp[-(1/2)(\theta/\theta_0)^2], \quad (6)$$

as long as the first order Taylor expansion term of these functions lead to the same result. Notice that the factor a of various forms may deviate from the conventional value (e.g. ~ 4), since the shape of the equal-arrival-time surface will be distorted due to the anisotropic distribution of the fireball energy. However, its time-dependence, if any, would be very small. Therefore, although it may influence the absolute values of the afterglow flux levels, such an effect does not change the blastwave dynamics in the viewing direction.

In principle, the jet configuration is time-dependent, due to the effects such as the energy redistribution and the lateral expansion. In the lab frame, the causally connected region subtends an angle of $\Delta\theta \sim c_s t'/R = c_s/c\Gamma(\theta)$, where $t' = R/c\Gamma(\theta)$ is the comoving time since the explosion along the θ direction, c_s is the expansion speed, which may be either the relativistic speed of sound $\sim c/\sqrt{3}$ (Rhoads 1999), or simply the speed of light (Sari et al. 1999). As $\Gamma(\theta_v) \gg 1/\theta_v$, the line-of-sight direction is causally disconnected from other regions, so the dynamics evolves essentially independently, so that the description in (3) holds. When the blastwave decelerates so that the line-of-sight bulk Lorentz factor $\Gamma(\theta_v)$ drops close to and below $1/\theta_v$, the dynamics along the line-of-sight starts to change, and $\bar{\Gamma}(\theta_v)$ will deviate from the $\propto t^{-3/8}$ dependence. There are several effects that play a role. First, as $1/\Gamma(\theta_v)$ exceeds θ_v, the observer starts to feel the energy deficit due to the drop of the energy distribution, i.e. deviation of the power law (1), on the other side of the jet axis. Although the calculation of $\bar{\epsilon}(\theta_v, t)$ is no longer straightforward, this deficit effect should mimic that in the uniform jet model as long as k is not too flat, say, $k > 1.5$. Second, the anisotropic jet has a trend to resume the isotropic shape. As $\Gamma(\theta_v) \sim 1/\theta_v$, the viewing direction starts to connect the jet axis causally. The energy outflow from the cone defined by θ_v becomes prominent, and this equivalently decreases $\bar{\epsilon}(\theta_v)$ in the viewing direction. In the meantime, the initial material within the θ_v cone starts to spread into a wider cone, and the observer would feel a stronger deceleration, although the global sideways expansion will become evident only when $\Gamma(\theta_v)$ drops below $1/\Theta$. All these effects tend to steepen the afterglow lightcurve, although the degree of steepening is unclear without detailed numerical dynamical calculations. In any case, in the asymptotic phase of sideway expansions, R is essentially a constant, and one would eventually have $\bar{\Gamma}(\theta_v, t) \propto t^{-1/2}$ (from $t \sim R/\bar{\Gamma}^2$, Rhoads 1997, 1999; Sari et al. 1999). In this regime, the temporal indices of the lightcurves in the various spectral regimes would follow closely the same predictions as in the uniform jet models (Sari et al. 1999; Rhoads 1999). For example, in the slow-cooling regime (which is usually the case after the viewing or "jet" break), for spectral regimes both below and above the cooling frequency, the asymptotic spectral flux is $F_\nu \propto t^{-p}$, where p is the power-law index of the electron number distribution. For reasonable values of p (e.g. ~ 2.2), this is consistent with several GRB afterglow observations. The above discussion should also hold for other distribution functions

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[1]Strictly speaking, the observer will see a smaller half cone on the close side of the jet axis, and a larger half cone on the far side to the axis, due to different Lorentz factors in different directions. This will modify the integral limits in (3), but does not influence the conclusion in (4) and the relevant discussions.

[2]The largest "jet" angle in F01 is 0.411, and the approximation is good within 3%.

such as (6), mainly because eventually all the initial configurations will be smeared out. However, to address the lightcurves properly within different models, including the relevant gradual transition between asymptotic regimes, a detailed dynamical description and numerical calculation is necessary, and we postpone this to a future work.

We have shown that the present model can reproduce the key features of the on-axis uniform jet model, with an arbitrary k value as long as it is not too flat. The next question is whether the model can also retain the merit of a standard energy reservoir invoked in the conventional jet model. In principle, one does not have to fulfill this constraint, but just wishes so for the sake of elegance. By definition, the total energy in a fireball with an energy distribution given by (1) is

$$E_{\rm tot} = 2\pi \int_0^\Theta \epsilon(\theta) \sin\theta d\theta \simeq 2\pi \int_{\theta_m}^\Theta \epsilon_0 \theta^{1-k} d\theta. \quad (7)$$

For $k < 2$ and $\Theta \gg \theta_m$, we get

$$E_{\rm tot} \simeq \frac{2\pi}{2-k} b^{2-k} \epsilon(\theta_v) \theta_v^2, \quad (8)$$

where we have parameterized $\Theta = b\theta_v$. We can see that the quantity $2\pi\epsilon(\theta_v)\theta_v^2$ (which is essentially the E_γ of F01, or E_0 of PK01 and P01) is quasi-invariant, if k and $E_{\rm tot}$ are constant (or have a small scatter). The only extra scatter is introduced through the scatter of b, which is introduced by the scatter of θ_v (assuming the same Θ for all GRBs). However, for the index $(2-k)$ this scatter is greatly reduced if k is not much smaller than 2. This is another reason why we require, say, $k > 1.5$, in (2). A smaller Θ can also reduce the b scatter. Notice that the b scatter tends to raise $E_{\rm tot}$ in GRBs with smaller θ_v's (and hence larger b's), which seems to be helpful to reduce the E_0 scatter in PK01. An important implication of equation (8) satisfying such a constraint is that the total energy reservoir is standard, but the absolute value need no longer necessarily be several times 10^{50} ergs, but would depend on the value of k and the typical value of b. Given reasonable values, $E_{\rm tot}$ could be one order of magnitude higher than that of F01 and PK01, but this could be still well accommodated within conventional central engine models (Mészáros , Rees & Wijers 1999). The closer k approaches 2, the larger the standard energy reservoir one requires. At $k = 2$, equation (8) should be modified in a form containing a logarithmic term, and the energy requirement is the highest (see discussions in Rossi, Lazzati & Rees 2001). Also the scatter of θ_m must be very small for $k = 2$, while for $k < 2$, the actual value of θ_m is not important. For $k \geq 2$, generally $E_{\rm tot}$ (eq.[7]) can not be expressed in terms of $\epsilon(\theta_v)\theta_v^2$, since most of the energy is distributed at small angles. The standard energy budget argument no longer holds[3]. A quasi-universal beaming configuration as well as a standard energy reservoir is however in general obtained if the requirement (2) is satisfied for an energy distribution such as (1).

For other forms of energy distributions, a standard energy reservoir is also attainable. For example, for the Gaussian distribution (6), one has $E_{\rm tot} \sim \epsilon_0 \theta_0^2$.

3. DISCUSSION

We have shown that an off-axis anisotropic jet with an energy distribution with angle given by equation (1) (or other forms such as [6]), is able to reproduce the key observational features of a conventional on-axis uniform jet model, e.g. such as producing a "jet break" signature in the light curve. The novelty here is that the achromatic break time t_b in the broadband afterglow lightcurves no longer corresponds to the time when the relativistic beaming angle is equal to the jet half opening angle, θ_j. Rather, it corresponds to the time when the relativistic beaming angle is roughly equal to the observer's viewing angle θ_v relative to the jet axis. In this model, the broad distribution of t_b in the data is no longer due to the intrinsic scatter of the jet opening angles among different bursts, but is attributed to the distribution of the observer's lines of sight. For a power law energy distribution (1) (or a Gaussian energy distribution [6]), if the constraint (2) is satisfied, all the GRBs may have a quasi-universal beaming configuration, besides a quasi-standard energy reservoir. We deem this to be a more elegant picture than the conventional on-axis uniform jet model. In addition, the homogeneous nature of the conventional model is more idealized, and the present inhomogeneous model is likely to be a closer representation of what could be expected in nature.

The predictions of this inhomogeneous model for the afterglow lightcurves are not completely equivalent to those of the uniform jet model. The key difference should occur around the "jet break" time. Our model should give a more gradual variation at the break than the uniform jet model, which assumes a sharp drop off at the jet edge. The so far sparsely studied sideways expansion effect in an anisotropic jet may further complicate the problem. The shape of the break should also depend on the angular energy distribution function and some unknown parameters, such as k. Detailed modeling is necessary in order to address these questions. In any case, the gradual break expected in our model is not inconsistent with several well studied afterglow lightcurves, and some simulations have shown that the conventional jet models usually also give gradual and smooth jet breaks (e.g. Panaitescu & Mészáros , 1999; Moderski, Sikora & Bulik 2000; Huang et al. 2000). Both models are compatible with the present data, but this situation may change as better data becomes available and as more detailed simulations are performed.

Recently, Rossi et al. (2001) have independently discussed the power-law model (1) in more detail. They plotted the afterglow lightcurves for the $k = 2$ case which mimic those of the on-axis uniform jet model, and also discussed the more general cases of $k \neq 2$. Here we have presented a general analytical argument, showing that the blastwave dynamics at the line of sight is identical to the uniform jet model in the asymptotic regime for a locus of models of the general form of equation (1), as long as k is not too flat. With this particular form of the angular dependence of the energy, in order to have a standard, finite energy reservoir for all bursts one requires the constraint (2). The upper end $k \lesssim 2$ of the constraint (2) ensures that the total energy can be expressed in terms of $\epsilon(\theta_v)\theta_v^2$ and

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[3]However, if θ_m is not too small (e.g. a not very small fraction of θ_v), the case $k \gtrsim 2$ could still retain the feature of a standard, finite (but even larger) energy reservoir.

does not diverge. (However, the case $k \gtrsim 2$ could also have the same virtue if θ_m is not too small compared with θ_v). The lower end of the constraint, $k > 1.5$, ensures that the scatter introduced by θ_v is not too large, and that the energy-deficit effect at the other side of the jet is not too small. We have also found that the main features in the power-law model are also applicable to some other angular energy distributions, e.g. such as the Gaussian form (6).

For ease of discussion, we have here assumed that the upper limit of validity of the assumed angular distribution is $\Theta > 2\theta_v$. This is to avoid that the observer feels the energy deficit beyond Θ before the relativistic beaming angle exceeds θ_v. Indeed, if $\Theta < 2\theta_v$, $\bar{\Gamma}(\theta_v)$ starts to deviate from the value predicted by the adiabatic law $\propto t^{-3/8}$ after it is less than $(\Theta - \theta_v)^{-1}$. In this regime, the upper limits for θ-integration in the numerators of both (3) and (4) should be replaced by Θ. Thus the maximum correction factor with respect to the $\Theta > 2\theta_v$ case is a factor of $\Theta/2\theta_v$. Even for $\Theta = \theta_v$ (i.e., the line of sight marginally cuts the jet edge), the deviation is at most a factor of $1/2$. We therefore conclude that the Θ effect may in most cases not be important. The main reason is that the large angles contribute a small portion of the total energy in the beam due to the distribution of the form (1).

In our model, the "isotropic" luminosity function will be determined by the assumed angular distribution, $N(\epsilon)d\epsilon = N(\theta)d\theta \propto \sin\theta d\theta \propto \theta d\theta$ (the latter being for small θ). From equations (1), (6) and substituting ϵ by L, we get the luminosity function predictions in our model, e.g.,

$$N(L)dL \propto L^{-1-2/k}dL \qquad (9)$$

for the power-law model, and

$$N(L)dL \propto L^{-1}dL \qquad (10)$$

for the Gaussian model. To test these luminosity functions, redshift measurements are needed. Using only the bursts for which optical redshifts have been determined so far, e.g. as compiled in F01, PK01, the above luminosity

distributions are not consistent. However, this discrepancy could be due to small number statistics (~ 20 in all or ~ 10 on each side of the mid-point). The fact that the small sample size or other selection effects related to the afterglow detections could lead to a spurious inconsistency is also suggested, for example, by the clear deficit of low luminosity (e.g. possibly due to large viewing angle) bursts at higher redshifts ($z > 1$) in F01's data set. Alternatively, some other distance indicators have been proposed, such as spectral time-lags, e.g. Norris, Marani & Bonnell 2000, or variability measures, e.g. Fenimore & Ramirez-Ruiz 2002, Reichart et al. 2001 (and interpretations of these indicators in terms of the viewing angle have been discussed by, e.g., Salmonson & Galama 2002 and Norris et al 2002). If one accepts such distance indicators and their inferred redshifts at face value, the observational GRB luminosity function inferred for a much larger bursts sample (e.g. Schaefer, Deng & Band 2001) is not inconsistent with the theoretical distribution (9). And, using a different approach, Schmidt (2001) obtained a flatter observational GRB luminosity function, which over a large range of luminosities is compatible with the model distribution (10).

A natural consequence of this model is that the distribution of break times t_b, and hence the θ_v distribution, should be related to the statistical distribution of viewing angles and to the shape of the beam distribution. The present data and the preliminary calculations are not sufficient to draw firm constraints on parameters of such models. However, the comparison of such predictions or more detailed versions of them against future data in the *Swift* era should provide interesting constraints, as a larger quantity of more accurate redshift measurements become available.

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